Performance Equations for Compressible Flow Through Orifices and Other ΔP Devices: A Thermodynamics Approach

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Introduction

Orifices are among the oldest and most widely used devices for measuring flows, but economic pressures recently have dictated a careful evaluation of this measurement method. The basic orifice equations result from thermodynamic derivations. The expressions presented here provide insight for the usual orifice equations and reveal a new approach.

Derivation for Compressible Fluids

The starting point for deriving orifice equations is the first law of thermodynamics. In general terms, this expression is:

$$\Delta[u + ke + pe]_{sys} = \Sigma q + \Sigma w + \Sigma[h + ke + pe]_{mt} \quad (1)$$

where u is internal energy, ke is kinetic energy, pe is potential energy, q is energy transferred as heat, w is energy transferred as work, and h is enthalpy. All properties are per unit mass. The summation signs represent "in minus out." Subscripts sys and mt denote system and mass transfer, respectively. For our purposes, we now assume:

- Steady state flow
- Adiabatic flow
- No work
- Horizontal flow (this assumption is not necessary, but simplifies the algebra)
- Turbulent flow (flat profile)
- Circular cross section

Applying these assumptions to Eq. 1 and taking subscript-indicated condition *1* to be upstream and condition *2* to be downstream produces:

$$h_2 - h_1 + \frac{\langle \dot{z}_2^3 \rangle}{\langle \dot{z}_2 \rangle} - \frac{\langle \dot{z}_1^3 \rangle}{\langle \dot{z}_1 \rangle} = h_2 - h_1 + \frac{\dot{z}_2^2 - \dot{z}_1^2}{2} = 0 \quad (2)$$

where $\dot{z}$ is velocity. The mass flowrate, $\dot{m}$, in steady state flow is:

$$\dot{m} = \dot{z}A/v \quad (3)$$

where A is the cross-sectional area of the conduit and v is the volume per mass of the fluid.

The kinetic energy term in Eq. 2 becomes, in terms of Eq. 3,

$$\frac{\dot{z}_2^2 - \dot{z}_1^2}{2} = 8\left[\frac{\dot{m}Z_1RT_1}{\pi D_1^2 MP}\right]^2 \left\{\left[\frac{Z_2T_2P_1}{Z_1T_1P_2}\right]^2 \left[\frac{D_1}{D_2}\right]^4 - 1\right\}$$
$$= -\frac{8\dot{m}Z_1RT_1\Psi}{\pi D_1^2 MP} \quad (4)$$

where Z is the compressibility factor, M is the molar mass, R is the gas constant, T is temperature, P is pressure, D is diameter, and Ψ is a function of the temperature and pressure of the fluid at points 1 and 2, and of the geometry also at points 1 and 2. Two equivalent expressions for the enthalpy difference in Eq. 2 are:

$$h_2 - h_1 = \int_{s_1}^{s_2} T ds + \int_{P_1}^{P_2} v dP \quad (6)$$

$$h_2 - h_1 = \frac{R}{M}\left\{ {}_{T_2}\!\int_0^{P_2} \left[\frac{\partial Z}{\partial 1/T}\right]_{P,x} \frac{dP}{P} \right.$$
$$\left. - {}_{T_1}\!\int_0^{P_1} \left[\frac{\partial Z}{\partial 1/T}\right]_{P,x} \frac{dP}{P} + \int_{T_1}^{T_2} \left[\frac{C_p^{id}}{R}\right] \mathrm{dT} \right\} \quad (7)$$

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where s is entropy per mass, x is composition, and C_p^{id} is ideal gas specific heat per mole. Substitution of Eq. 4 and either Eq. 6 or Eq. 7 into Eq. 2 provides an expression for $\dot{m}$.

Equations 6, 4, and 2 form the basis for the usual orifice expressions after utilizing further definitions. One definition is

$$ {}_1f_2 = \int_{s_1}^{s_2} T\,ds - {}_1q_2 = \dot{z}_1^2 \phi_1(\beta, N_R, \ldots)/2 \qquad (8) $$

where ${}_1f_2$ is the dissipative (friction) term, and ϕ is an empirical function of geometry, flow characteristics, fluid properties, and probably other terms. The arguments listed are $\beta = d/D$ where d is the orifice diameter and N_R is the Reynolds number. The ${}_1q_2$ term is energy transferred as heat along path 1 → 2. Thus, Eq. 6 becomes

$$ h_2 - h_1 = {}_1f_2 + \int_{P_1}^{P_2} v\,dP = \dot{z}_1^2\phi_1/2 + \int_{P_1}^{P_2} v\,dP \qquad (9) $$

The integral remaining in Eq. 9 is particularly vexing. Although the enthalpy is a path-independent function, Eq. 9 forces the path to be the actual physical path through the orifice. Because this path is irreversible, calculation of the integral is not tractable, necessitating an additional definition

$$ \int_{P_1}^{P_2} v\,dp = F_1^2 Y_1^2 v_1 [P_2 - P_1] \qquad (10) $$

where F and Y are empirical factors that account for irreversibility and for fluid expansion.

This approach produces an expression for mass flow rate

$$ \dot{m} = \left[\frac{\pi^2}{8R}\right]^{1/2} \Phi_1 Y_1 d^2 \left\{\frac{MP_1}{Z_1T_1}[P_2 - P_1]\right\}^{1/2} \qquad (11) $$

where Φ_1 is a collection of dissipative and expansion effects:

$$ \Phi_1 = \frac{F_1}{\beta_1^2[\Psi - \phi_1]^{1/2}} = \Phi_1(\beta, N_R, T, P, x, \ldots) \qquad (12) $$

For compressible fluids, the ZTP ratio term in Ψ is not unity. Also because Ψ and ϕ_1 appear in an additive manner, the Y term cannot account totally for the expansion effect in Ψ. Equation 11 corresponds formally with the usual orifice expressions such as appear in Hall et al. (1983), Miller (1983), AGA Report Number 3, and numerous texts on fluid mechanics.

Equations 7, 4, and 2 produce a new performance equation

$$ \dot{m} = \frac{\pi D_1^2 P_1}{Z_1T_1}\left[\frac{M}{8R\Psi}\right]\left\{ {}_{T_2}\!\int_0^{P_2}\left[\frac{\partial Z}{\partial 1/T}\right]_{P,x}\frac{dP}{P} - {}_{T_1}\!\int_0^{P_1}\left[\frac{\partial Z}{\partial 1/T}\right]_{P,x}\frac{dP}{P} + \int_{T_1}^{T_2}\left[\frac{C_p^{id}}{R}\right]dT\right\}^{1/2} \qquad (13) $$

The most striking feature of this equation is that it has no explicit dissipative term. In fact, the entire effect of the orifice appears in the pressure and temperature changes. The temperature must change for compressible fluids because the first law limit is isenthalpic operation and the second law limit is isentropic operation. Isothermal operation lies outside these bounds.

Finally, Eqs. 11 and 13 provide insight into the problem of correlating the discharge coefficient. Equating the two expressions produces

$$ \Phi_1\beta_1^2Y_1 = \frac{\left\{ {}_{T_2}\!\int_0^{P_2}\left[\frac{\partial Z}{\partial 1/T}\right]_{P,x}\frac{dP}{P} - {}_{T_1}\!\int_0^{P_1}\left[\frac{\partial Z}{\partial 1/T}\right]_{P,x}\frac{dP}{P} + \int_{T_1}^{T_2}\left[\frac{C_p^{id}}{R}\right]dT\right\}^{1/2}}{Z_1T_1\Psi[P_2 - P_1]/P_1} \qquad (14) $$

Problems with past correlations for discharge coefficients include ignoring the temperature effect, assuming that Y_1 can account for all expansion effects, and trying to correlate friction.

Discussion

Equation 13 has some advantages for measuring flow rates. Because it is a function only of fluid properties and observables (with all dissipative effects contained in the measurable pressure and temperature changes), it is relatively easy to provide necessary correlations: a composition-dependent equation of state, and a temperature-dependent expression for C_p^{id}. Many relationships are available for these properties and new ones are becoming available.

Measurement of pressure and temperature changes must occur at locations where they reflect the fluid properties. Pressure measurements in eddies and temperature measurements that indicate the pipe temperature are not acceptable. The stream diameters, D_1 and D_2, must also be available at the tap locations. With proper measurement of pressure and temperature changes, the physical appearance of the flow impedance is not relevant. In fact, damage to the impedance which does not alter D_1/D_2 has no effect upon flow rate calculation.

Example

Consider the sensitivity of flow rate calculation to ΔT using various configurations for an orifice in a 250 mm pipe. The gas flowing through the line has a molar mass of 17.38, the temperature is 15.6°C, and the pressure is 15 bar (1,500 kPa). Ambient conditions are 15.6°C, 1 bar (100 kPa), with a gravitational acceleration of 980.665 cm/s². The equations for compressibility factor and specific heat are

$$ Z = 1 + [0.00042 - 0.3/T]P $$
$$ C_p^{id} = 7.00 - T/1{,}000 $$

Pipe taps with ΔP = 1,016 mm H₂O, ΔT = −0.0744°C, β = 0.6

	ΔT		
ΔP	−0.0750	−0.0744	0.0739
1,013	—	8.107	—
1,016	9.631	7.215	3.376
1,019	—	6.202	—

Flange taps with ΔP = 1,778 mm H_2O, ΔT = −0.4944°C, β = 0.6

ΔP	ΔT		
	−0.4950	−0.4944	−0.4939
1,775	—	7.245	—
1,778	7.244	7.239	7.233
1,781	—	7.237	—

Flange taps with ΔP = 400 in. H_2O, ΔT = −3.0278°C, β = 0.4

ΔP	ΔT		
	−3.0283	−3.0278	−3.0272
10,157	—	7.245	—
10,160	7.245	7.244	7.244
10,163	—	7.244	—

In this example, the flow rates in lbm/s appear within the $\Delta P/\Delta T$ grid. We have chosen ΔT = 0.0006°C as an achievable precision (using difference thermistors, for example). In each case, the base flow rate is nearly identical. Clearly, the pipe taps are not acceptable here becaue ΔT = 0.0006°C produces variations of up to 53%. Using flange taps with β = 0.6, the same ΔT produces variations of only 0.08% (a ΔT of 0.0006°C produces variations of 0.76%). Using flange taps with β = 0.4, the same ΔT produces variations of only 0.01% (0.1% for ΔT = 0.006°C). Clearly, a smaller β ratio would produce a higher overall ΔT and the sensitivity would be even less.

Notation

A = area
C_p^{id} = ideal gas specific heat at constant pressure
d = orifice diameter
D = pipe diameter
$_1f_2$ = friction in process 1 → 2
F = empirical factor for irreversibility
h = enthalpy per mass
ke = kinetic energy per mass
$\dot{m}$ = mass flow rate
M = molar mass
pe = potential energy per mass
P = pressure
q = heat per mass
R = gas constant
s = entropy per mass
T = temperature
u = internal energy per mass
v = volume per mass
w = work per mass
x = composition
y = empirical factor for expansion
$\dot{z}$ = velocity
Z = compressibility factor

Greek Letters

$\beta = d/D$
Ψ = defined by Eq. 4
ϕ = friction factor
Φ = defined by Eq. 12
$\langle\,\rangle$ = expectation value

Literature Cited

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